SAMPLE PAPER

TIME : 3 HRS. **MAX. MARKS : 80**

GENERAL INSTRUCTIONS :

- **All questions are compulsory.**
- **The question paper contains two parts A and B.**
- **Both Part-A and Part-B have internal choices.**
- **Part-A consist of two Sections - (I) and (II).**

  Section-(I) has 16 questions of 1 mark each. Internal choices is provided in 5 questions.

  Section-(II) has 4 questions on case study. Each case study has 5 case - based subparts out of which 4 has to be attempted carrying 1 mark for each subpart.
- **Part-B consist of three Sections - (III), (IV) and (V).**

  Section-(III) has 6 questions of 2 marks each. Internal choices is provided in 2 questions.
  Section-(IV) has 7 questions of 3 marks each. Internal choices is provided in 2 questions.
  Section-(V) has 3 questions of 5 marks each. Internal choices is provided in 1 question.

PART-A

SECTION-I

1. Terminating decimal expansion of $\frac{51}{1500}$ is in the form of $\frac{17}{2^n \times 5^m}$ then find (m + n).

2. If f(x) = ax + b; then find the zero of f(x)

OR

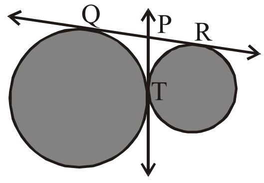
If the product of zeroes of the polynomial $ax^2 - 6x - 6$ is 4, find the value of 'a'.

3. Find value(s) of k for which quadratic equation $kx^2 - kx + 2 = 0$ has equal roots.

4. Find the 21^{st} term of an AP whose first two terms are –3 and 4.

5. Find the distance of point P(2, 3) from X-axis.

6. In figure, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If PT = 3.8 cm, then find the length of QR (in cm)



7. Find value of k if lines 3x + 2ky = 2 and 2x + 5y + 1 = 0 are parallel.

8. Find 25^{th} term of AP $-5, -\frac{5}{2}, 0 ; \frac{5}{2}$ ....

9. Find the number of cubes of side 2 cm which can be cut from a cube of side 4 cm.

OR

The surface area of a sphere is 616 sq cm. Find its radius $\left(\pi = \frac{22}{7}\right)$.

10. First and last term of an A.P. are 8 and 65 respectively and sum of all its terms is 730, find the number of terms.

OR

Find the tenth term of AP $\sqrt{2}, \sqrt{8}, \sqrt{18}$.....

11. Find the distance of point P(x,y) from origin.

12. Find the value of x, if the distance between the points (x, – 1) and (3, 2) is 5.

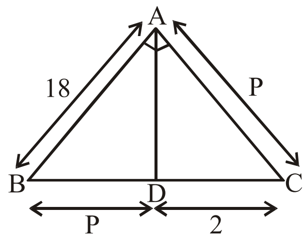
13. For what value of k will $\frac{7}{3}$ be a root of $3x^2 - 13x - k = 0$.

14. If sum of the squares of zeroes of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, find the value of k.

15. D and E are respectively the points on the sides AB and AC of a ΔABC such that AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm, show that DE || BC.

OR

In given figure, if $\Delta ADB \sim \Delta ADC$, then find the value of p.



16. Which term of the sequence 4, 9, 14, 19, ... is 124 ?

OR

If n^{th} term of an A.P. is (2n + 1), what is the sum of its first three terms?

SECTION-II

17. **Case study Based -1**

In a court-piece game of playing cards, there are four players. Pair of two-two persons are made partners. In a deck of 52 – playing cards, cards are distributed around the table clockwise in batches of 5 – 4 – 4 cards.

(a) The probability that the card drawn is jack of red colour is

(i) $\frac{25}{26}$ (ii) $\frac{1}{13}$ (iii) $\frac{1}{26}$ (iv) $\frac{5}{26}$

(b) The probability that card drawn is a face card is

(i) $\frac{1}{13}$ (ii) $\frac{2}{13}$ (iii) $\frac{3}{13}$ (iv) $\frac{4}{13}$

(c) The probability of a red colour card is

(i) $\frac{1}{2}$ (ii) $\frac{1}{26}$ (iii) $\frac{1}{52}$ (iv) None of these

(d) The probability that card drawn is from suite of clubs is

(i) $\frac{1}{2}$ (ii) $\frac{1}{3}$ (iii) $\frac{1}{4}$ (iv) $\frac{3}{4}$

(e) The probability that card is either king or queen is

(i) $\frac{3}{26}$ (ii) $\frac{1}{26}$ (iii) $\frac{3}{13}$ (iv) $\frac{2}{13}$

18. Case study Based -2

Some students went on excursion to Agra to Visit Taj Mahal. After taking a close look at monument, one of them told others that this monument has combination of solid figures. The main structure has a big central hemispherical dome and four other smaller hemispherical domes called "Chattri" around the bigger dome. There are four cylindrical pillars called "Minarets" at four corners around main structures.

(a) Find curved surface area of four cylindrical Minarets if their height is 14 m and base radius 2 m each.

(i) 352 m^2 (ii) 600 m^2 (iii) 704 m^2 (iv) None of these

(b) Find volume of air inside a Minaret of height 14 m and base radius 2 m.

(i) 176 m^3 (ii) 100 m^3 (iii) 208 m^3 (iv) 352 m^3

(c) What is the ratio of surface area of big central hemispherical dome of radius 21 m and sum of surface areas of four smaller hemispherical domes each of radius 7 m.

(i) 3 : 4 (ii) 3 : 2 (iii) 9 : 2 (iv) 9 : 4

(d) What will be the formula of total outer curved surface area of central dome if it has a cylindrical base of same radius 'r' and height 'h'.

(i) $2\pi rh$ (ii) $4\pi r^2 + 2\pi rh$ (iii) $2\pi r^2 + 2\pi rh$ (iv) None of these

(e) What is volume of air inside central dome along with cylindrical base. If common radius is 21 m and height of cylindrical portion is 3 m.

(i) 24000 m^3 (ii) 23562 m^3 (iii) 11781 m^3 (iv) None of these

19. Case study Based -3

A house of cards or card castle is a structure created by stacking playing cards on top of each other, often in shape of pyramid. To build a tower of cards 2 cards are placed against each other in an upside down position forming a triangular shape. Another triangle next to it and so on. Same process is repeated in next row and so on. A man formed such a house of cards with 41 such triangular shape as base, 39 in the next, 37 in next and so on. He made 16 such rows in this manner one above another.

(a) Number of triangles in the top row is

(i) 10 (ii) 11 (iii) 12 (iv) 9

(b) Difference of number of triangles in the 8^{th} and 13^{th} row from bottom is

(i) 10 (ii) 12 (iii) 15 (iv) 20

(c) The number of triangles in the 11^{th} row from the bottom is

(i) 10 (ii) 20 (iii) 21 (iv) 25

(d) Number of deck of cards each having 52 playing cards used by him if total cards he used was 832 is

(i) 5 (ii) 6 (iii) 8 (iv) 16

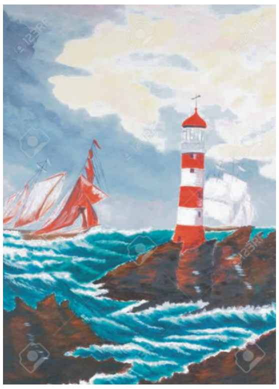
(e) The number of triangles in the 4^{th} row from the top is

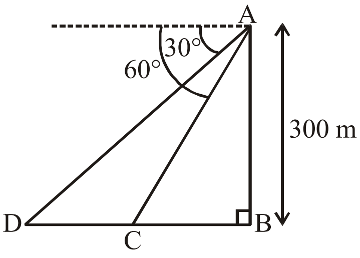
(i) 15 (ii) 16 (iii) 17 (iv) 18

20. Case study Based -4

A light house is a tower, builiding designed to emit light from a system of lamps and lenses to serve as a navigation for ships.

As observed from top of a light house 300 m high, the angle of depression of two ships coming towards the base of light house as 30° and 60° respectively.





(a) Distance of farther ship from base of light house

(i) $100\sqrt{3}$ m (ii) $200\sqrt{3}$ m (iii) $300\sqrt{3}$ m (iv) 300 m

(b) Distance of nearer ship from base of light house

(i) $100\sqrt{3}$ m (ii) $200\sqrt{3}$ m (iii) 300 m (iv) None of these

(c) Distance between the two ships

(i) 100 m (ii) 200 m (iii) $200\sqrt{3}$ m (iv) $100\sqrt{3}$ m

(d) Distance of farther ship from top of light house

(i) 100 m (ii) 300 m (iii) 400 m (iv) 600 m

(e) If speed of both ships are equal and nearer ship take 10 min to reach base of light house, in how many minutes farther ship reach base of light house

(i) 20 min (ii) 30 min (iii) 40 min (iv) 60 min

PART-B

SECTION-III

21. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

22. It is known that a box of 600 electric bulbs contains 12 defective bulbs. One bulb is taken out at random from this box. What is the probability that it is a non-defective bulb ?

23. Prove that $3 + 2\sqrt{5}$ is irrational, given that $\sqrt{5}$ is irrational.

OR

The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.

24. Is the following pair of linear equations consistent ? Justify your answer.

$2ax + by = a$, $4ax + 2by - 2a = 0$; $a , b \neq 0$.

25. A die is thrown once. Find the probability of getting :

(i) a prime number

(ii) a number lying between 2 and 6

26. In an A.P, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the A.P., where S_n denotes the sum of first n terms.

OR

How many terms of the A.P. 27, 24, 21, ... should be taken so that their sum is zero?

SECTION-IV

27. Prove that : $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

OR

Prove that $\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta\cos\theta$

28. In what ratio does the point $\left(\frac{24}{11}; y\right)$ divide the line segment joining the points P(2, –2) and Q(3, 7).

Also find y.

29. For which values of a and b does the following pair of linear equations have an infinite number of solutions ?

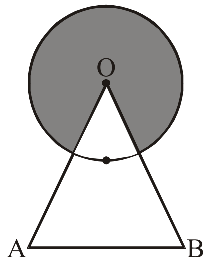
$2x + 3y = 7$

$(a - b)x + (a + b)y = 3a + b - 2$

OR

Solve for x and y : $\frac{2}{x} + \frac{3}{y} = 13$; $\frac{5}{x} - \frac{4}{y} = -2$; $x, y \neq 0$

30. Find the area of shaded region shown in the given figure where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



31. Show that $\sqrt{2}$ is an irrational number.

32. For the following grouped frequency distribution find the mode :

Class	3 – 6	6 – 9	9 – 12	12 – 15	15 – 18	18 – 21	21 – 24
Frequency	2	5	10	23	21	12	3

33. ABC is a triangle in which AB = AC and D is any point in BC. Prove that $AB^2 - AD^2 = BD \cdot CD$.

SECTION-V

34. Prove that : $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = 1 + \tan A + \cot A = 1 + \sec A \operatorname{cosec} A$.

OR

If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, prove that $(m^2 - n^2)^2 = 16\,mn$

35. ABC is a right triangle, right angled at B. AD and CE are two medians drawn from A and C respectively.

If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm. Find the length of CE.

36. PQ is chord of length 16 cm, of a circle a radius 10 cm. The tangents at P and Q intertsect at a point T. Find the length of TP.

MATHEMATICS SAMPLE PAPER

ANSWER AND SOLUTIONS

PART-A

SECTION-I

1. $\frac{51}{1500} = \frac{17}{500} \Rightarrow \frac{17}{5^3 \times 2^2} = \frac{17}{2^n \times 5^m}$

$\therefore$ m = 3, n = 2

Hence m + n = 5

2. f(x) = ax + b

$\Rightarrow ax + b = 0 \Rightarrow x = -\frac{b}{a}$

OR

$\frac{c}{a} = 4$

$\therefore \frac{-6}{a} = 4$

$\Rightarrow a = -\frac{3}{2}$

3. $b^2 - 4ac = 0$

$k^2 - 4(k)(2) = 0$

$k^2 - 8k = 0$

k(k – 8) = 0

k = 0 Not possible

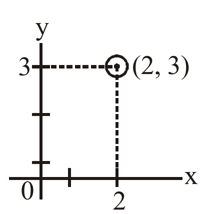
k = 8

4. a = –3

d = 4 – (–3) = 7

$a_{21} = a + 20d = -3 + 20(7) = -3 + 140 = 137$

5. Distance from x-axis is 3 units



6. PT = PR = PQ = 3.8 cm

(Tangents from external point)

$\therefore$ QR = 3.8 + 3.8 = 7.6 cm

7. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{3}{2} = \frac{2k}{5} \Rightarrow k = \frac{15}{4}$

8. a = –5

$d = -\frac{5}{2} + 5 = \frac{5}{2}$

$a_{25} = a + 24d \Rightarrow -5 + 24 \times \frac{5}{2}$

$\Rightarrow -5 + 60 = 55$

9. Volume of big cube = n × volume of small cube

$(4)^3 = n \times (2)^3$

$\frac{64}{8} = n$

$\Rightarrow n = 8$

OR

$4\pi r^2 = 616$

$4 \times \frac{22}{7} \times r^2 = 616$

$\Rightarrow r^2 = \frac{616 \times 7}{4 \times 22}$

$\Rightarrow r^2 = 49$ or r = 7 cm

10. $S_n = \frac{n}{2}(a + a_n)$

$730 = \frac{n}{2}(8 + 65)$

$\Rightarrow 730 = \frac{n}{2} \times 73$

$\therefore$ n = 20

OR

$\sqrt{2}; 2\sqrt{2}; 3\sqrt{2}$ ......

$a_{10} = a + 9d$

$= \sqrt{2} + 9\sqrt{2}$

$= 10\sqrt{2} = \sqrt{200}$

11. OP = $\sqrt{(x-0)^2+(y-0)^2} = \sqrt{x^2+y^2}$ units

12. Let P(x, –1) and Q(3, 2) be the given points. Then,

PQ = 5

$\Rightarrow \sqrt{(x-3)^2+(-1-2)^2} = 5$

$\Rightarrow (x-3)^2 + 9 = 5^2$

$\Rightarrow x^2 - 6x + 18 = 25$

$\Rightarrow x^2 - 6x - 7 = 0$

$\Rightarrow (x-7)(x+1) = 0$

$\Rightarrow x = 7$ or, $x = -1$

13. Putting $x = \frac{7}{3}$ in $3x^2 - 13x - k = 0$

$3\left(\frac{7}{3}\right)^2 - 13\left(\frac{7}{3}\right) - k = 0$

$\frac{49}{3} - \frac{91}{3} - k = 0$

$k = -\frac{42}{3} = -14$

14. Let α, β be the zero of the polyhomial $f(x) = x^2 - 8x + k$. Then,

$\alpha + \beta = -\left(\frac{-8}{1}\right) = 8$ and $\alpha\beta = \frac{k}{1} = k$

It is given that

$\alpha^2 + \beta^2 = 40$

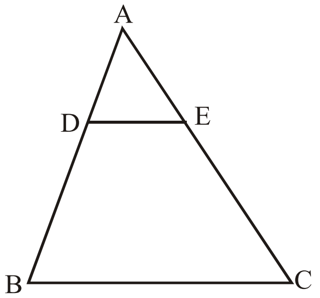
$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = 40$

$\Rightarrow 8^2 - 2k = 40$

$\Rightarrow 2k = 64 - 40$ [∴ α + β and αβ = k]

$\Rightarrow 2k = 24 \Rightarrow k = 12$

15. We have,



AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm.

∴ BD = AB – AD = (5.6 – 1.4) cm = 4.2 cm

and, EC = AC – AE = (7.2 – 1.8) cm = 5.4 cm

Now, $\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ and $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

Thus, DE divides sides AB and AC of ΔABC in the same ratio, Therefore, by the converse of Basic Proportionality Theorem, we have

DE || BC

OR

Since, ΔADB ~ ΔADC

$\frac{BD}{DC} = \frac{AB}{AC}$

$\frac{P}{2} = \frac{18}{P}$

$P^2 = 36$

P = 6

16. Clearly, the given sequence is an A.P. with first term a (= 4) and common difference d (= 5)

Let 124 be the nth term of the given sequence. Then,

$a_n = 124$

$\Rightarrow a + (n-1)d = 124 \Rightarrow 4 + (n-1) \times 5 = 124$

$\Rightarrow 5n = 125 \Rightarrow n = 25$

Hence, 25th term of the given sequence is 124.

OR

$a_1 = 3$, $a_3 = 7$

$s_3 = \frac{3}{2}(3 + 7) = 15$

SECTION-II

17. $P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total outcomes}}$

(a) Favourable → 2 cards

Total → 52 cards

$P(E) = \frac{2}{52} = \frac{1}{26}$ option (iii)

(b) Favourable → 12 cards

Total → 52 cards

$P(E) = \frac{12}{52} = \frac{3}{13}$ option (iii)

(c) Favourable → 26 cards

Total → 52 cards

$P(E) = \frac{26}{52} = \frac{1}{2}$ option (i)

(d) Favourable → 13 cards

Total → 52 cards

Proability $= \frac{13}{52} = \frac{1}{4}$ option (iii)

(e) Favourable → 8 cards

Total → 52 cards

Proability $= \frac{8}{52} = \frac{2}{13}$ option (iv)

18. (a) Curved surface area

$= 4 \times 2\pi rh$

$= 4 \times 2 \times \frac{22}{7} \times 2 \times 14$

$= 704 \text{ m}^2$ option (iii)

(b) Volume $= \pi r^2 h$

$= \frac{22}{7} \times 2 \times 2 \times 14$

$= 176 \text{ m}^3$ option (i)

(c) $\frac{\text{Surface Area of Big Dome}}{4 \times \text{Surface Area of Smaller Dome}}$

$= \frac{2\pi R^2}{4 \times 2\pi r^2}$

$= \frac{21 \times 21}{4 \times 7 \times 7}$

$= \frac{9}{4}$ option (iv)

(d) $2\pi r^2 + 2\pi rh$ option (iii)

(e) Volume $= \frac{2}{3}\pi r^3 + \pi r^2 h$

$= \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 + \frac{22}{7} \times 21 \times 21 \times 3$

$= 19404 + 4158 = 23562 \text{m}^3$ option (ii)

19. (a) $a_n = a + (n-1)d$

$= 41 + (16-1)(-2)$

$= 41 - 30 = 11$ option (ii)

(b) $a_8 - a_{13} = a + 7d - (a + 12d)$

$= -5d$

$= -5(-2) = 10$ option (i)

(c) $a_{11} = 41 + (11-1) \times (-2)$

$= 41 - 20 = 21$ option (iii)

(d) Number of decks $= \frac{\text{Total cards}}{\text{Cards in a deck}}$

$= \frac{832}{52} = 16$ option (iv)

(e) $a_n = a + (n-1)d$

$a = 11$

$n = 4$

$d = 2$

$a_n = 11 + 3(2) = 17$ option (iii)

20. (a) In ΔABD $\tan 30° = \frac{300}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{300}{BD}$

$\Rightarrow BD = 300\sqrt{3}\text{ m}$

option (iii)

(b) In ΔABC $\tan 60° = \frac{300}{BC}$

$\Rightarrow \sqrt{3} = \frac{300}{BC}$

$\Rightarrow BC = \frac{300}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 100\sqrt{3}\text{ m}$ option (i)

(c) Distance between two ships

$300\sqrt{3} - 100\sqrt{3} = 200\sqrt{3}\text{ m}$ option (iii)

(d) In ΔABD $\sin 30° = \frac{AB}{AD}$

$\frac{1}{2} = \frac{300}{AD}$

$\Rightarrow AD = 600$ m option (iv)

(e) For $100\sqrt{3}$ m time taken 10 min

For $300\sqrt{3}$ m time taken

$\frac{10}{100\sqrt{3}} \times 300\sqrt{3} = 30$ min option (ii)

PART-B

SECTION-III

21. In 60 minutes $\rightarrow 360°$

$\therefore$ In 5 minutes $\rightarrow \frac{360}{60} \times 5$

$= 30°$

Area $= \frac{\pi r^2 \theta}{360} = \frac{22}{7} \times \frac{14 \times 14 \times 30}{360}$

$= \frac{154}{3} = 51\frac{1}{3}$ cm^2

22. Out of 600 electric bulbs one bulb can be chosen in 600 ways.

$\therefore$ Total number of elementary events = 600

There are 588 (= 600 – 12) non-defective bulbs out of which one bulb can be chosen in 588 ways.

$\therefore$ Favourable number of elementary events = 588

Hence, P(Getting a non-defective bulb)

$= \frac{588}{600} = \frac{49}{50} = 0.98$

23. Let us assume, to the contrary, that $3 + 2\sqrt{5}$ is rational. That is, we can find coprime integers a and b ($b \neq 0$) such that $3 + 2\sqrt{5} = \frac{a}{b}$, $b \neq 0$,

a, b $\in$ I

Therefore, $\frac{a}{b} - 3 = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$

$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2} = \sqrt{5}$

Since a and b are integers, we get $\frac{a}{2b} - \frac{3}{2}$ is rational, and so $\frac{a-3b}{2b} = \sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction has arisen because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

OR

$\ell = 850$

$b = 625$

$h = 475$

longest rod = HCF (850, 625, 475)

$= 5^2 = 25$ cm

24. Yes,

Here, $\frac{a_1}{a_2} = \frac{2a}{4a} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-a}{-2a} = \frac{1}{2}$

$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$ The given system of equations is consistent.

25. We have, the total number of possible outcomes associated with the random experiment of throwing a die is 6 (i.e. 1, 2, 3, 4, 5, 6).

(i) Let E denotes the event of getting a prime number.

So, favourable number of outcomes = 3(i.e., 2, 3, 5)

$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$

(ii) Let E be the event of getting a number lying between 2 and 6.

$\therefore$ Favourable number of elementary events (outcomes) = 3 (i.e., 3, 4, 5)

$$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$$

26. $S_5 + S_7 = 167$

$$\frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167$$

$\Rightarrow 24a + 62d = 334$

or $12a + 31d = 167$ ....(i)

$S_{10} = 235$

$\Rightarrow 5(2a + 9d) = 235$

or $2a + 9d = 47$ ....(ii)

Solving (i) and (ii), wet get

$a = 1, d = 5$

Here A.P. = 1, 6, 11, ...

OR

Here, $a = 27$, $d = -3$, $S_n = 0$

$$\therefore \frac{n}{2}[54 + (n - 1)(-3)] = 0$$

$\Rightarrow n = 19$

SECTION-IV

27. LHS $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$

$$= \sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{2}{\cos\theta} = 2\sec\theta = \text{RHS}$$

Hence proved

OR

We have,

$$\text{LHS} = \frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$$

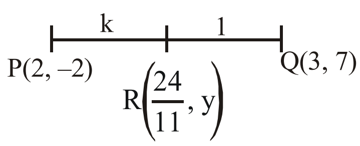
$$\Rightarrow \text{LHS} = \frac{\cos^3\theta}{\cos\theta-\sin\theta} - \frac{\sin^3\theta}{\cos\theta-\sin\theta}$$

$$\Rightarrow \text{LHS} = \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$$

$$\Rightarrow \text{LHS} = \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)}{\cos\theta-\sin\theta}$$

$\Rightarrow$ LHS = $1 + \sin\theta\cos\theta$ = RHS

28.



Let R divides PQ in ratio k : 1

By section formula co-ordinates of 'R' are

$$\frac{m_1x_2+m_2x_1}{m_1+m_2} ; \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$\left(\frac{3k+2}{k+1} ; \frac{7k-2}{k+1}\right)$$

$$\frac{3k+2}{k+1} = \frac{24}{11}$$

$33k + 22 = 24k + 24$

$9k = 2$

$k = \dfrac{2}{9}$ Ratio $\to$ 2 : 9

$$y = \frac{7k-2}{k+1} \Rightarrow \frac{7\times\frac{2}{9}-2}{\frac{2}{9}+1}$$

$$y = \frac{-4}{11}$$

29. We have, $2x + 3y = 7$ ..... (i)

$(a - b)x + (a + b)y = 3a + b - 2$ ..... (ii)

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = 7$

and $a_2 = a - b$, $b_2 = a + b$, $c_2 = 3a + b - 2$

For infinite number solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

Now, $\frac{2}{a-b} = \frac{3}{a+b}$

$\Rightarrow 2a + 2b = 3a - 3b$

$\Rightarrow 2a - 3a = -3b - 2b$

$\Rightarrow -a = -5b$

$\therefore a = 5b$ ...... (iii)

Again, we have

$\frac{3}{a+b} = \frac{7}{3a+b-2} \Rightarrow 9a + 3b - 6 = 7a + 7b$

$\Rightarrow 9a - 7a + 3b - 7b - 6 = 0$

$\Rightarrow 2a - 4b - 6 = 0 \Rightarrow 2a - 4b = 6$

$\Rightarrow a - 2b = 3$ ...... (iv)

Putting $a = 5b$ in equation (iv), we get

$5b - 2b = 3$ or $3b = 3$ i.e., $b = \frac{3}{3} = 1$

Putting the value of b in equation (iii), we get

$a = 5(1) = 5$

Hence, the given system of equations will have an infinite number of solutions for

$a = 5$ and $b = 1$.

OR

Given equations

$\frac{2}{x} + \frac{3}{y} = 13$ .....(1)

$\frac{5}{x} - \frac{4}{y} = -2$ .....(2)

Let $\frac{1}{x} = u$, $\frac{1}{y} = v$

From (1) and (2)

$2u + 3v = 13$ ......(3)

$5u - 4v = -2$ ......(4)

Multiplying equation (3) from 5 and equation (4) by 2 and subtract them

$10u + 15v = 65$

$10u - 8v = -4$

$-\quad +\quad +$

$23v = 69$

$v = 3$

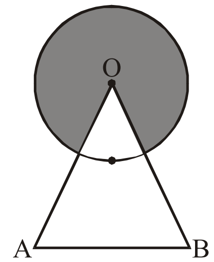
From (3) $2u + 3.3 = 13$

$2u = 4$

$u = 2$

Thus, $x = \frac{1}{2}$, $y = \frac{1}{3}$

30.



$\angle AOB = 60°$

Area of shaded region = Area of major sector

$= \frac{300}{360} \times \frac{22}{7} \times (6)^2 = 94.29 \text{ cm}^2$

31. Let $\sqrt{2}$ be rational

i.e $\sqrt{2} = \frac{p}{q}$ ($q \neq 0$, p and q are co-prime)

Squaring $2 = \frac{p^2}{q^2}$

$\therefore q^2 = \frac{p^2}{2}$ .....(1)

If 2 divides p^2 ; then 2 divides p i.e.

2 is factor of p

Let $p = 2k$ ....(2)

Putting value of p from (2) in (1)

$q^2 = \frac{4k^2}{2}$

$k^2 = \frac{q^2}{2}$

If 2 divides q^2 then 2 divides q i.e. 2 is factor of q.

$\Rightarrow$ 2 is common factor of p and q

Which is contrary to our assumption

Hence $\sqrt{2}$ is irrational.

32. We observe that the class 12 – 15 has maximum frequency. Therefore, this is the modal class.

We have,

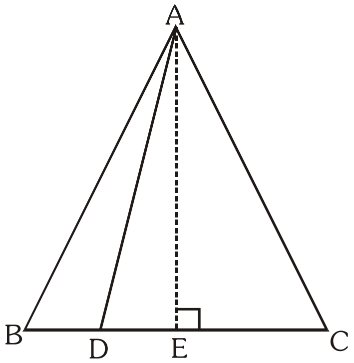
$\ell = 12$, $h = 3$, $f = 23$, $f_1 = 10$ and $f_2 = 21$

$$\therefore \text{Mode} = \ell + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$\Rightarrow \text{Mode} = 12 + \frac{23-10}{46-10-21} \times 3$$

$$\Rightarrow \text{Mode} = 12 + \frac{13}{15} \times 3 = 12 + \frac{13}{5} = 14.6$$

33. Draw AE ⊥ BC



In Δ AEB and ΔAEC, we have

AB = AC

AE = AE [Common]

∠B = ∠C [∵ AB = AC]

∠AEB = ∠AEC [Each 90°]

∴ ΔAEB ≅ ΔAEC [by AAS congruence]

$\Rightarrow$ BE = CE [by cpct]

Since ΔAED and ΔABE are right triangles right angled at E. Therefore,

$AD^2 = AE^2 + DE^2$ and $AB^2 = AE^2 + BE^2$

$\Rightarrow AB^2 - AD^2 = BE^2 - DE^2$

$\Rightarrow AB^2 - AD^2 = (BE + DE)(BE - DE)$

$\Rightarrow AB^2 - AD^2 = (CE + DE)(BE - DE)$

$\Rightarrow AB^2 - AD^2 = CD \cdot BD$

$\Rightarrow AB^2 - AD^2 = BD \cdot CD$

Hence proved

SECTION-V

34. $$\text{LHS} = \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{1}{\tan A(1-\tan A)}$$

$$= \frac{\tan^2 A}{\tan A - 1} - \frac{1}{\tan A(\tan A - 1)}$$

$$= \frac{\tan^3 A - 1}{\tan A(\tan A - 1)}$$

using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$\frac{(\tan A - 1)(\tan A + 1 + \tan^2 A)}{\tan A(\tan A - 1)}$$

$$\frac{\tan A + 1 + \tan^2 A}{\tan A}$$

$$= \frac{1}{\tan A} + \frac{\tan^2 A}{\tan A} + \frac{\tan A}{\tan A}$$

$= \cot A + \tan A + 1$

Hence proved

$$\text{Now } 1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$$

$$= 1 + \frac{1}{\cos A \sin A}$$

$= 1 + \sec A \operatorname{cosec} A$

Hence proved

OR

$m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta$

$- \sin^2\theta + 2\tan\theta\sin\theta$

$m^2 - n^2 = 4\tan\theta\sin\theta$

$\therefore (m^2 - n^2)^2 = 16\tan^2\theta\sin^2\theta$ ......(1)

Now, $16mn = 16(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)$

$= 16[\tan^2\theta - \sin^2\theta]$

$$= 16\left[\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta\right]$$

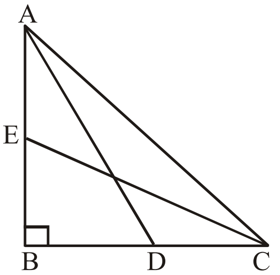
$$= 16\sin^2\theta\left[\frac{1}{\cos^2\theta} - 1\right]$$

$= 16\sin^2\theta[\sec^2\theta - 1]$

$16mn = 16\sin^2\theta\tan^2\theta$ ....(2)

$\therefore (m^2 - n^2)^2 = 16\,mn$

35. In ΔABD ; $\angle B = 90°$



$\therefore AD^2 = AB^2 + BD^2$

$$AD^2 = AB^2 + \left(\frac{BC}{2}\right)^2$$

$$AD^2 = AB^2 + \frac{BC^2}{4} \quad .....(1)$$

In ΔBEC

$CE^2 = BC^2 + BE^2$

$$CE^2 = BC^2 + \left(\frac{AB}{2}\right)^2$$

$$CE^2 = BC^2 + \frac{AB^2}{4} \quad ......(2)$$

equation (1) and (2)

$$AD^2 + CE^2 = AB^2 + \frac{BC^2}{4} + BC^2 + \frac{AB^2}{4}$$

$$AD^2 + CE^2 = \frac{5}{4}(AB^2 + BC^2)$$

$$AD^2 + CE^2 = \frac{5}{4}AC^2 \quad [AC^2 = AB^2 + BC^2]$$

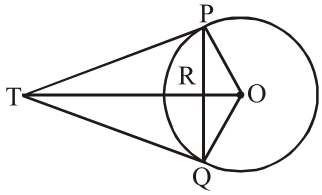
$$\left(\frac{3\sqrt{5}}{2}\right)^2 + CE^2 = \frac{5}{4} \times 25$$

$$CE^2 = \frac{125}{4} - \frac{45}{4}$$

$CE^2 = 20$

$CE = \sqrt{20} = 2\sqrt{5}$ cm

36.



Given PQ = 16 cm

Radius = 10 cm

To find : TP

Solution : Join OP and OQ

In ΔOTP and ΔOTQ

OP = OQ (Radius)

TP = TQ (Tangents from external point)

OT = OT (common)

$\therefore \Delta OPT \cong \Delta OQT$ (SSS)

$\angle POT = \angle QOT$ (cpct) ....(i)

In ΔOPR and ΔOQR

OP = OQ

$\angle POR = \angle QOP$ (from (i))

OR = RO

$\Delta OPR \cong \Delta OQR$ (SAS)

So, $PR = RQ = \frac{1}{2} \times 16 = 8$ cm .....(ii)

$\angle ORP = \angle ORQ = 90°$ .....(iii)

(By cpct)

In ΔOPR

$OR^2 = OP^2 - PR^2$

$OR^2 = 100 - 64$

$OR = 6$

In ΔTRP $TR^2 = TP^2 - 64$ .....(iv)

In ΔTOP

$OT^2 = TP^2 + (10)^2$

$(TR + OR)^2 = TP^2 + 100$

$(TR + 6)^2 = TP^2 + 100$

$TR^2 + 12TR + 36 = TP^2 + 100$

$TP^2 - 64 + 12TR + 36 = TP^2 + 100$

(using (iv))

$12TR = 128$

$$TR = \frac{32}{3} \text{ cm}$$

from (iv) $\left(\frac{32}{3}\right)^2 = TP^2 - 64$

$$TP^2 = \frac{1024}{9} + 64$$

$$TP^2 = \frac{1024+576}{9} = \frac{1600}{9}$$

$$TP = \frac{40}{3} \text{ cm}$$